For office use only

T1 ____________________

T2 ____________________

T3 ____________________

T4 ____________________

For office use only

F1 ____________________

F2 ____________________

F3 ____________________

F4 ____________________

2017

20th Annual High School Mathematical Contest in Modeling (HiMCM) Summary Sheet

Team Control Number: 8393

Problem Chosen: A

Summary

For problem A, our team was tasked with the creation of an aerial light show in a large city's annual outdoor festival. The show would be divided into three separate displays, two of which were provided in the task and one that was up to our team's creativity. The requested displays included a ferris wheel and dragon, and we chose to model a solar system in the final display. To create the displays, we debated between creating a grid of drones or setting exact positions for them in each display. For accuracy, we chose to set specific courses and positions for each drone in the model. We began determining the mathematical parameters of each display with a simple sketch. We sketched first the ferris wheel, then the dragon and solar system and plotted points where each of the drones would be placed. From this, we determined 400 drones would be sufficient in modeling all of the displays. We considered safety requirements including the necessary distance between each drone and the orientation of the launch area. With a 5-foot safety distance between each drone, there is room for any sort of interference of the wind or slight inconsistencies in the technology. Our display is designed to be safe for an audience. In terms of launching our drones, we decided to lay each drone on the ground to avoid any problems associated with stacking. We represented each drone a 1-foot-radius circle to ensure that no two drones would be too close together. For the most efficient use of ground space, we use a hexagonal packing method. We then decided to use nine distinct drone launches to ensure that no two drones are closer than five feet when in the air. Then, we modeled the points and molded our displays to fit within our safety guidelines. There are two different options regarding costs, which vary whether the city decides to rent or buy the drones. With the recommended method of buying the drones, the one-time cost would total roughly 100,000 dollars. For renting the drones, a cost of 36,000 would be spent each year or for each event with drones. As for the necessary air space, we found the area used in each display, added a buffer zone and followed FAA's requirements.

Letter to the Mayor

Dear Mayor,

We have looked into the option of adding an outdoor drone light show to our annual festival. We think that adding this element would be an excellent addition. Not only is this idea unique, it is also impressive and safe for viewers.

From the designs of a ferris wheel, dragon, and solar system, we concluded that 400 drones would be sufficient. To ensure safety for viewers, we decided on a 5 ”safety zone” surrounding each drone in the air.

For the first display, a ferris wheel was requested. We examined many models of ferris wheels and concluded that a standard model included 20 spokes. An outer circle frames the spokes, and there is an inner circle that the spokes all lead to. From there, we had to determine how to distribute the 400 drones amongst the spokes and inner/outer circles of the ferris wheel. For the outer circle, we determined that we should put 40 drones, one drone at the outer edge of each spoke and one in between each spoke. To evenly disperse the remaining drones, we represented each spoke with 17 drones. For the inner circle of the ferris wheel, we used 10 drones instead of 20 to ensure airspace safety. We split the drones of the ferris wheel into an anterior and posterior plane. Every other drone on the spokes will be placed in the opposite plane to ensure airspace safety. The center drone of the model would be suspended in a middle plane. We mathematically pathed out the rotation of the drones so that the display would look like a rotating ferris wheel. For visuals, we determined three separate light models of the ferris wheel. The show will begin with four different groups of drones. Red, blue, yellow and green will represent four fractions of the spokes. These will be accompanied by purple around the outer circle of the ferris wheel. The next light model of the ferris wheel will be an alternating pattern of purple and yellow, with red drones on the outer circle in between these spokes. The final light model of the ferris wheel will have a pink outer circle of drones. The half of each of the spokes that is closest to the center will be light blue, and the outer halves will be blacked out. Each light model of the ferris wheel will be present for one minute, or two total revolutions. The entire ferris wheel section of the show will last 3 minutes in order to keep the viewer’s entertained. In the transition to the next display, we will light off random drones in random patterns and colors as they travel to their next position.

We will then transition to the dragon display. We based our model off of the Chinese dragon. Groups of drones will be split into the dragons body, head, tail and fire breath. We used circles following the path of sin(x) to illustrate the dragons body. In each of the 25 circles making up the body, 8 drones will be evenly dispersed. 120 drones were used in the model of the head. 37 drones made up the tail, and 36 of the remaining drones were used in the dragon’s ”fire breath.” To animate the dragon’s figure, it will follow a wave-like pattern that will intrigue the viewers. As for the colors of the dragon display, red, white and yellow will be utilized as in an actual Chinese dragon. The drones on the spine of the dragon and the fire will project yellow. The undercarriage and eyes of the dragon will be projected in white. The remaining parts of the head, body and tail of the dragon will appear red. Once the transition period of random lighting passes following the ferris wheel, the dragon display will be present for 3 minutes. Then, another transition period of one minute will occur as the drones prepare for the final display.

The final display of the light show will be a model of the solar system. We designed a solar system with the sun and its 8 planets. To scale the grandiose of the sun, we used 100 yellow drones for its representation. The next planet in revolution, Mercury, will be represented by 10 orange drones in a spherical formation. Venus will be represented by 28 white drones, and Earth will be represented by a conglomeration of 28 green/blue drones. The next planet in orbit is Jupiter, shown by 52 orange/white drones. Saturn will also be represented by 52 drones, but they will be orange. 20 white drones will be used to represent the rings of Saturn. Uranus will be represented by 42 green drones. Lastly, Neptune will be represented by 42 purple drones. The total number of drones used in this model will be 396, so four will be remaining and unlit. The drones will remain in their spherical formations to represent the planets as the participate in a circular orbit around the ”sun” formation. The drones in the sun formation will remain hovering as the rest of the planets orbit. The solar system display will last for three minutes. Following the display of the solar system, there will be one minute allotted for the drones to safely return to the launch pad.

There are two options for use of these drones: buying from Intel or renting. If the city was to buy these drones, we would likely need to purchase programming software from Intel. Although the official price has not been released, we estimate that Intel's ”Shooting Star” drones will cost around 200 dollars each. The price of 400 drones would likely be around 80,000 dollars. With operating costs and possible software, we can estimate about 100,000 in expenditures.

As for the renting option of these drones, Intel may offer a program to rent their drones for a small period of time. We believe Intel is likely to charge about 20 percent of the cost of purchasing the drones when renting. Therefore, to rent the drones would cost around 20,000 dollars. We assume that with a software program and operating costs, the total cost of renting these drones would be about 36,000 dollars.

We recommend the buying option of the drones. We would be able to run our own displays and use the drones whenever the city wished. It may become difficult for the drones to arrive for each event from a distant location. Renting the drones as little as three times would amount to the total cost of buying the drones. In the long run, the buying option would be more cost-effective. Unlike a fireworks show, the drones could be reused.

A safety concern with using drones for the light show is that a drone would lose power or control. If a drone is to fall or cause a collision with another drone, we want to ensure that the audience will be completely void of danger. In the unlikely event that two drones collide, the caging around their propellers should keep them from falling. We inputted a 5-foot safety distance between each drone in all of our displays to ensure that if the programming is off, there is a reasonable leeway to avoid disaster. We plan to put the launch pad well in front of the audience in case a drone goes rogue. Our show will occur above the launch pad at a safe distance form the audience. If, in practice or the actual show, a drone somehow falls near the audience, no serious injuries should occur. The drones are made of flexible plastic and foam that prevent any sort of safety hazard. The base of the display will be 100 feet above the ground in order to make sure they do not come into contact with any obstacles like power lines. We will also follow the FAA regulations, including never letting the display go over 400 feet. Therefore the air space we need is 1500 feet wide and long, and 350 feet tall.

Drone Cluster as Sky Light Displays

HiMCM team 8393

November 18, 2017

Contents

1 Restatement of the Problem

Our city is contemplating the idea of adding an outdoor aerial light show to our annual festival. The mayor has tasked our team with determining the parameters involved with using drones for three sky displays. The displays will include a ferris wheel, a dragon, and an additional image chosen by us. Once we have chosen the third display, we are responsible with calculating the number of drones required for the show. We must determine the initial location for each drone that will result in a static image. Then, we must describe the flight paths of each drone in a mathematical context. There must be a clear distinction of the paths that will animate the image. We must explain the required launch area, air space, safety and duration of the light show. Finally, we must provide a report for the mayor with our recommendation. After analyzing the advantages and disadvantages of the aerial light show, we should have a clear choice of whether or not to make this addition to the festival.

2 Assumptions

Initially, we assumed that the drones created by Intel are one square foot each, similar to other drones.(Source 6)

We assume that the drones will require a 5 foot radius sphere of empty airspace around them for safe flight, this would allow for a total of 4 feet between the edges of each drone. This would ensure that there is plenty of space in case of a drone failure.

We also assume that, when the drones are in the sky, they behave like point light sources. Intel's drones emit light in all directions below them, and they are relatively small, so it is safe to assume that a drone's light will never be pointed in the wrong direction or blocked by another drone.(Source 2)

Intel's Shooting Star drones reportedly have a flight time of about 20 minutes.(Source 2)

Another assumption, given that the price of Intel's "Shooting Star" drones has not been released, is that the drones will cost roughly 200 dollars each. We gathered this from other drone prices and looked at drones with similar flight times, size, and quality of Intel's. (Source 6)

We assume that the drones can safely and accurately move at 15 miles per hour, which is similar to other drones.(Source 6)

We assumed the shape of the dragon, and that it would be a Chinese dragon.

We assumed that the city would want a large enough show to be seen from a large distance and by a large area.

We also assumed the city would want to avoid any conflict with FAA and aviation activity.

We assumed the city would have a large enough area to host this drone show, and that this area is owned by the city, therefore it would not cost anything to rent the space.

3 Variables

Variables were created in addition to assumptions, allowing for more flexibility.

1. Total number of drones
2. Speed of the drones
3. Cost of the drones
4. Safety zone of the drones
5. Safety air space
6. Size of Launch Area
7. Ferris Wheel

Number of drones

Number of drones on the outside circle of the ferris wheel

Number of spokes on the ferris wheel

Number of drones on the spokes of the ferris wheel

Radius of the ferris wheel

8. Dragon

Number of drones

Number of circles in the dragon's tail

Number of drones per circle

Number of drones in the dragon's face

Drones for special effects

9. Solar System

Number of drones

Number of planets

Distribution of drones amongst planets/sun

Number of orbits

4 Drone Launching

We will start by attempting to launch 100 drones. In terms of launching space, we have had multiple ideas. Given that each drone is roughly one foot square, and that each drone will require at least 5 feet of empty space in each direction for safe flight, we have decided to arrange drones on the ground in such a way that they are no less than two feet apart. We will then launch the drones in sequence, with multiple distinct mass launches occurring to ensure that the drones are far enough from each other when in flight.

Initially, we tried arranging the drones on the ground in a simple grid with two feet between each drone. This was relatively efficient, as the grid took up a 20 foot by 20 foot square of ground space with an area of 1600 square feet.

We also considered a system that would involve stacking drones on top of each other using some sort of guide to hold them in place. However, we decided not to use this system, as we think that it has too many risks. For example, if the first drone to launch ended up setting the rest of the stack askew, the launch could be ruined. Another possible scenario would be if the drones were stacked improperly, then a drone would try to take off with other drones still on top of it. This would likely be catastrophic. With all drones on the ground, even if they were placed improperly and two took off next to each other, there would still be some space between them which would allow for safe flight.

However, we decided that it would be more sensible to represent each drone as a one-foot radius circle. This way, we can pack the drones as efficiently as possible, in a hexagonal pattern (known as "hexagonal packing"), and our "two feet between the center of each drone" requirement will be fulfilled as long as the circles never intersect. This hexagonal packing design could be represented by a grid, with every second row being moved over by half a column. Due to the nature of this isometric grid system, our ground layout will now have dimensions of

$$(x+1)feet \times (y \times \frac{sqrt3}{2})feet$$

where x is the width and y is the height of a standard grid.

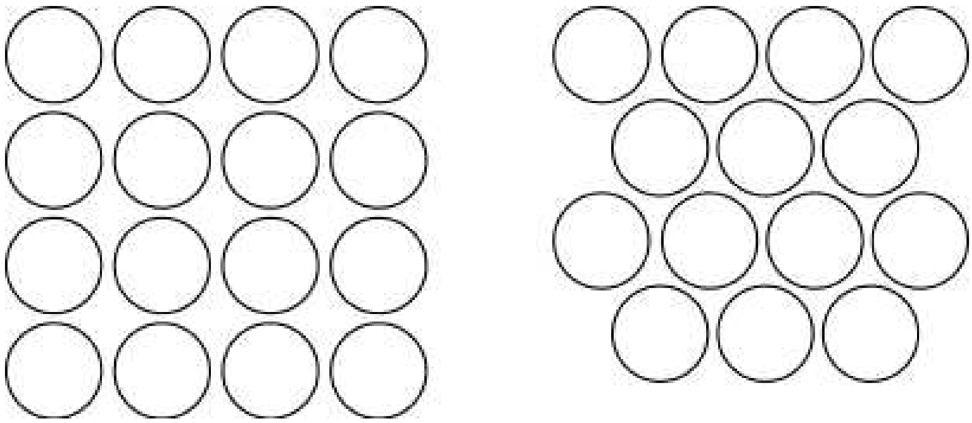

Figure 1: Square vs. Hexagonal Drone Packing

This system is very easily expandable, and could be used with three drones as readily as with a thousand.

We have opted for nine distinct launches, as we think that this will allow for an efficient and safe takeoff method. Each "launch number" is indicated on the chart above. Our system works by using 3 x 3 groups of drones, which are launched one at a time in each group. Multiple groups are arranged into an isometric grid, thus leading to a modular system that could theoretically launch thousands of drones in just nine distinct launches.

1	2	3	1	2	3	1	2	3	1
4	5	6	4	5	6	4	5	6	4
7	8	9	7	8	9	7	8	9	7
1	2	3	1	2	3	1	2	3	1
4	5	6	4	5	6	4	5	6	4
7	8	9	7	8	9	7	8	9	7
1	2	3	1	2	3	1	2	3	1
4	5	6	4	5	6	4	5	6	4
7	8	9	7	8	9	7	8	9	7
1	2	3	1	2	3	1	2	3	1

Table 1: Drone launch sequences on a 10 x 10 grid.

Since we will be using 400 drones in our show, we will have a "grid" of 20 drones by 20 drones. The area occupied will be roughly 1420 square feet.

$$(20drones \times 2ft + 1ft) \times (20drones \times \frac{\sqrt{3}}{2}) = 1420drones$$

A standard grid would require 1600 square feet, so this is a significant improvement.

As the drones rise from their position in the hexagonally-packed grid on the ground, they will all go to their separate positions. Based on their launch group number, the drones will fly out in groups. Each group will contain 44-45 drones and will fly to separate points, at which time the drones will spread out to form our first display: the Ferris wheel.

5 Ferris Wheel

We started the ferris wheel model with 100 drones. We wanted a circle and 20 spokes with one drone in the center. Initially we tried to use a fixed grid of drones to model the ferris wheel, similar to how a computer monitor works. With 100 drones, that gave us a 10 by 10 grid. We realized that with a grid, we would not be able to make an accurate circle without an unreasonable number of drones.

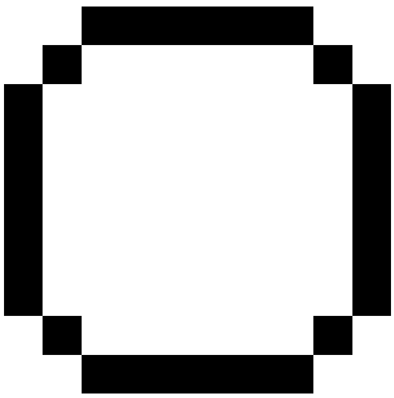

Figure 2: Circle plotted on a 10 by 10 graph

In order to get a decent circle we would need a 25 by 25 grid.

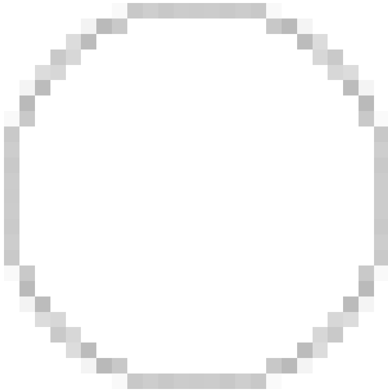

Figure 3: Circle plotted on a 25 by 25 graph

The additional drones needed to complete the square grid rather than a circle pattern are unnecessary. Also, it would be difficult to make use of all the drones if we had to disable the lights on some to fit the ferris wheel in the grid.

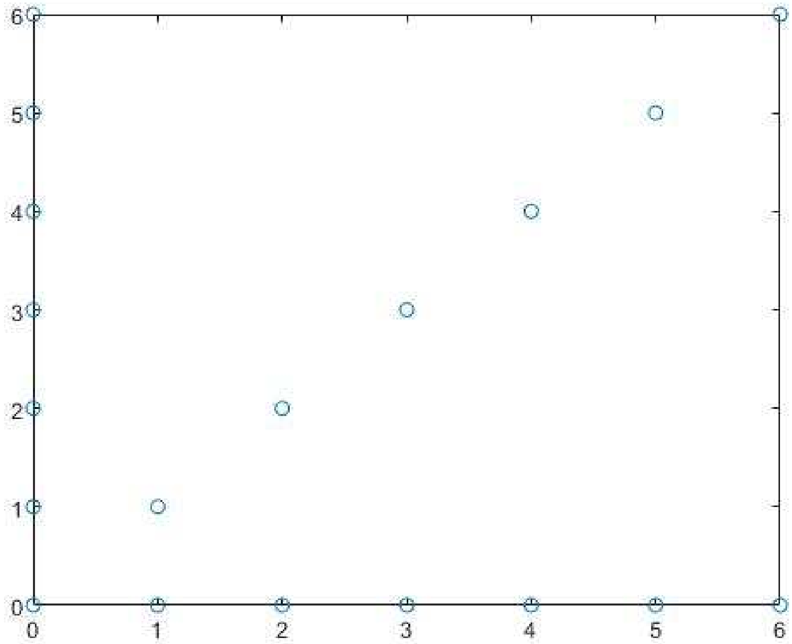

Figure 4: Plotted points of ferris wheel spokes in a grid.

Next, we overlayed points on an image of a standard ferris wheel (Source 8). We placed points on the outside circle where each of the spokes intersected. There were 20 spokes in our initial model, so therefore there were 20 points on the outside circle. Then, we put one point in the center where a drone would be placed. We began plotting points on the spokes in a circular formation until there were 100 total plotted points. We concluded that each spoke would be able to have 5 points disregarding the center point.

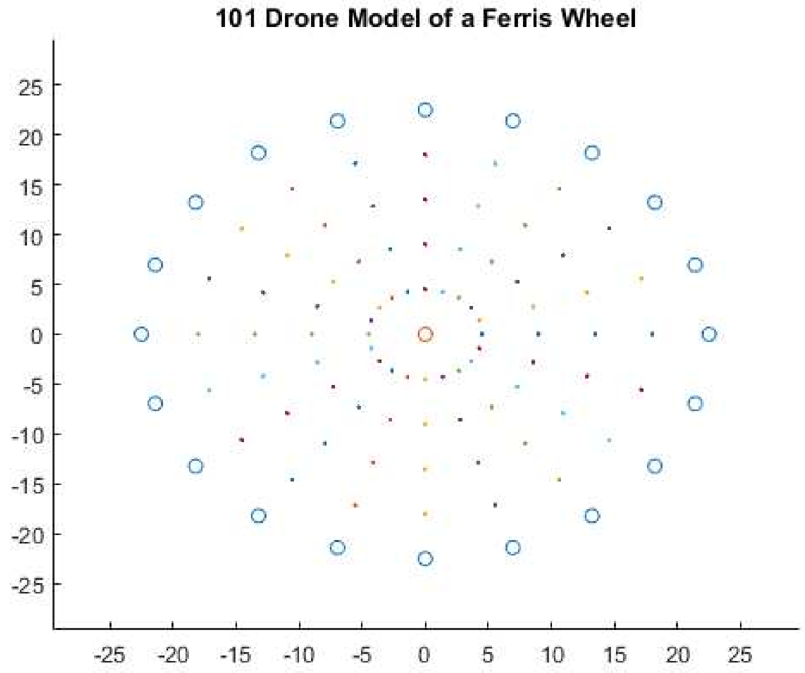

Figure 5: 101 points plotted on ferris wheel.

We realized that for a light show viewed in a large city, 100 drones would not provide a super vibrant image. We then examined the ferris wheel outline using the same technique with 400 points, or drones. Using 400 drones, there would be 20 on the outside circle, and the remaining 380 drones would be split among the spokes. We did not include the drone in the center for our initial calculation. This would result in about 17 drones set at an even distance apart on each spoke. If 17 drones are placed on each spoke, there will be 40 excess drones that we could use as replacement in case of unseen accident, for special effects, and possibly for a safety perimeter in the sky.

We had concluded that with a 5-foot safety distance between each drone, the circle would not appear full unless there were additional drones between each spoke intersection on the outside circle. We decided to place our excess drones between each of the intersections so there are 40 drones total in the outside circle. Since the innermost circle of the ferris wheel has a radius of 5 ft, we decided to eliminate 10 drones so that none of the safety zones would be compromised. We also added one drone in the center point. This left a total of 29 drones for replacement for special effects, and a safety perimeter in the sky.

We debated whether or not to layer two planes of drones or have the entire image displayed in one plane. When the drones are all displayed on a single plane, they would appear further apart due to our limitations with space and safety. With the drones split among two separate planes, one in front of the other, the image would appear more condensed and the lines would be more definite.

The drones will be split into the anterior and posterior planes. We also created a middle plane where, in the center of the ferris wheel, we placed the center drone. We decided to place one drone in the center in just one plane because if we had placed a center drone in the anterior and posterior planes, then the two planes would have to be at least five feet apart. We placed the center drone in a separate plane instead of in the front or the back because it would be more visually pleasing. For the drones on the circumference of the circle, those at intersections with spokes will remain in the anterior plane, and the drones in between these intersections will be in the posterior plane. On the spokes, every other drone will be in the posterior plane. There center drone will be in the middle plane.

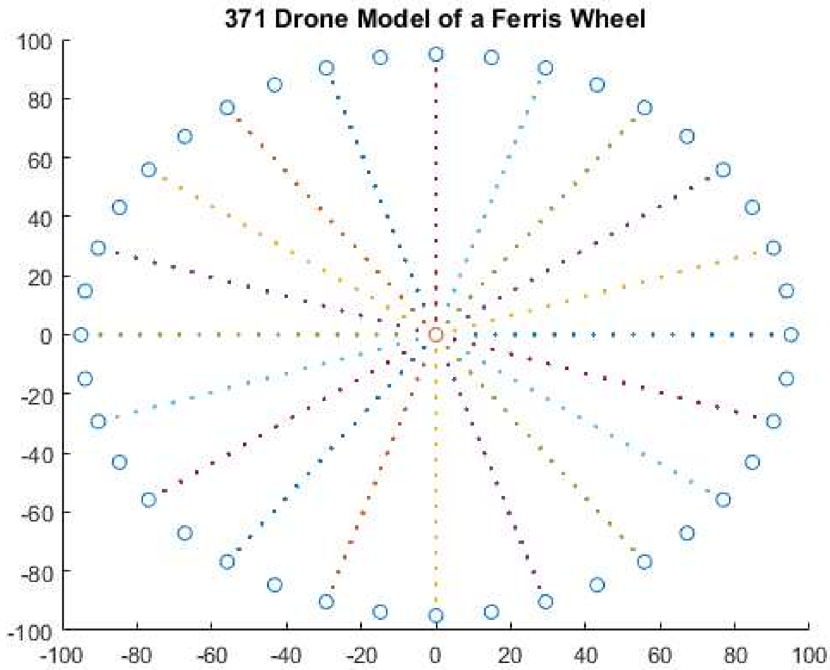

Figure 6: Final model of the ferris wheel using 371 drones.

Now that we have the initial positions for all the drones required in the Ferris wheel, we can plan the paths of the drones from takeoff to their positions in the Ferris wheel. Based off of their launch groups, the drones will fly in straight lines in groups to various positions and then split off to form the shape. The positions in the Ferris wheel that the drones from each group will occupy are shown below. Because of their flight paths and the order in which they launch, the groups of drones will never cross paths, which results in little to no risk of crashing.

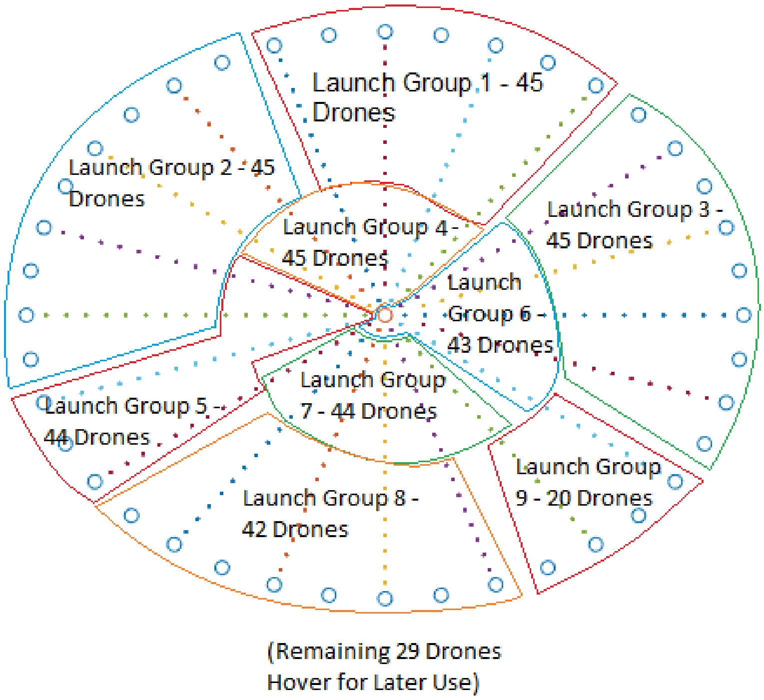

Figure 7: Model of drone positions based on launch group.

Now that we found a position for the drones, we are able to animate them. We decided that the drones will move in the revolution of the ferris wheel at 2 revolutions each minute. The ferris wheel's diameter is 190 feet.

$$Circumference = 190\pi = 596.9 feet/rev$$

We assumed that the drones can move at 15 miles per hour. In order to find out how many revolutions per minute the drones could fly, we used the equation below.

$$\frac{15 miles/hr \times 5280 ft/mile}{60 min} = 1,320 ft/min$$

Once we knew the speed of the drones, 1,320 feet per minute, we figured out how many revolutions per minute the ferris wheel was capable of.

$$\frac{1,320 ft/min}{596.9 ft/rev} = 2.211 rev/min$$

Since the ferris wheel is able to revolve just over 2 times per minute, we decided it would revolve exactly 2 times per minute in our light show. To figure out the angular velocity of the drones we used this equation:

$$\frac{rev}{min} \times \frac{1 min}{60 sec} \times \frac{2\pi rad}{rev} = \frac{rad}{sec}$$

With our calculations inputted in the equation, it appears as so:

$$\frac{2rev}{1min} \times \frac{1min}{60sec} \times \frac{2\pi rad}{1rev} = \frac{\pi}{15} rad/sec$$

We figured the ferris wheel show would be present for 3 minutes, totaling 6 revolutions.

Now that we had the speed and revolutions that the drones could take, we found equations to mathematically show the path of the animation. The drones would fly in a circle to simulate a ferris wheel. The path would be able to easily explained by polar equations to create a circle. What we found was to path out (x,y):

$$(r \times cos(v \times t), r \times sin(v \times t))$$

r is the radius at which the drones will be at, v is the angular velocity the drones will be moving at, which is $\frac{\pi}{15}$ rad/sec, and t is time in seconds. For example the outer most circle, which has a radius of 95 ft, would be:

$$(95 \times cos(\frac{\pi}{15} \times t), 95 \times sin(\frac{\pi}{15} \times t))$$

After figuring out the animations, we decided on interesting patterns for the light show. The show will begin by separating the spokes into groups of five and assigning them different colors: red, blue, yellow, and green. The outer drones would be purple.

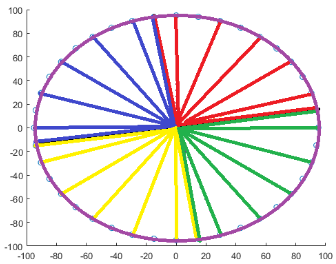

Figure 8: 1st Display of drone lights

The second display would consist of each spoke and corresponding circle drone alternating purple and yellow. Each outside circle drone that isn't attached to a spoke would be red.

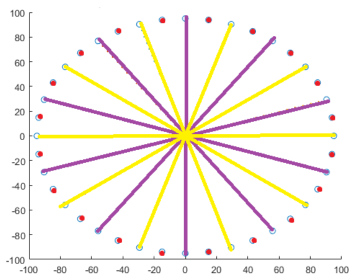

Figure 9: 2nd Display of drone lights

The third display would have the outer circle drones be pink. The inner half of drones in the spokes would be a light blue while the outer half of drones in the spokes would be blacked out.

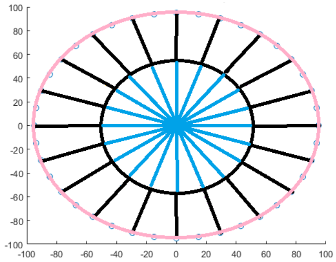

Figure 10: 3rd Display of drone lights

6 Dragon

We chose the Chinese Dragon as the dragon for our light display.

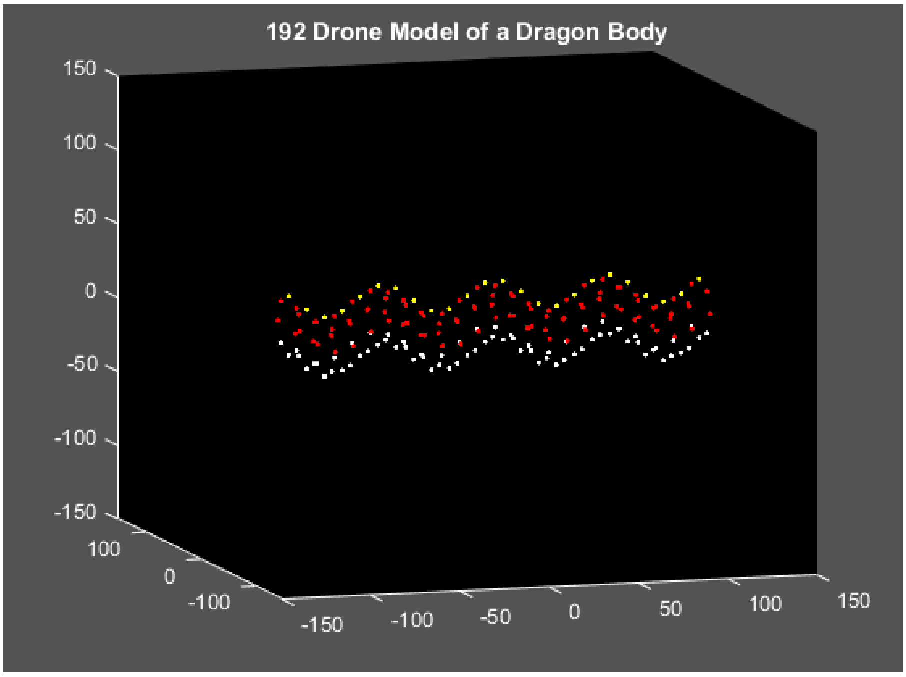

Figure 11: Model of the dragon's body using 192 drones

The tail of the Chinese dragon appears similar to the graph of sin(x), so we thought it would be fitting to mathematically program its animation using this function. We simplified the shape of the dragon's tail into that of a cylinder following the path of sin(x). We determined that we would have 25 points on the line $z = 10sin(x/10)$ since this gave a good amplitude and a good period. Each point would be used as the center of a circle containing 8 drones. The distance that each drone would be away from the sine wave would be $\Delta y = rcos(\theta)$ and $\Delta z = rsin(\theta)$, where r is the radius of each circle. We determined that r would be 20 and θ would step by increments of $\pi/4$ in the range $0 \leq \theta < 2\pi$ in order to make circles containing 8 drones each. The 3D plot of the drones making up the body of the dragon can be seen in Figure 13. The number of drones used by the body would be given by the following equation:

$$8drones \times 25circles = 200drones$$

Out of our total of 400 drones, there will be 200 drones remaining for use for the rest of the dragon. We drew a basic outline of the dragons head and plotted points. There was a total of 28 points on the outline we designed. In order to make it look 3-D, we placed three layers of these outlines at a distance of 20 feet from each other. We then scaled the points to make the back of the head 40 feet tall so that it would fit with the body of the dragon.

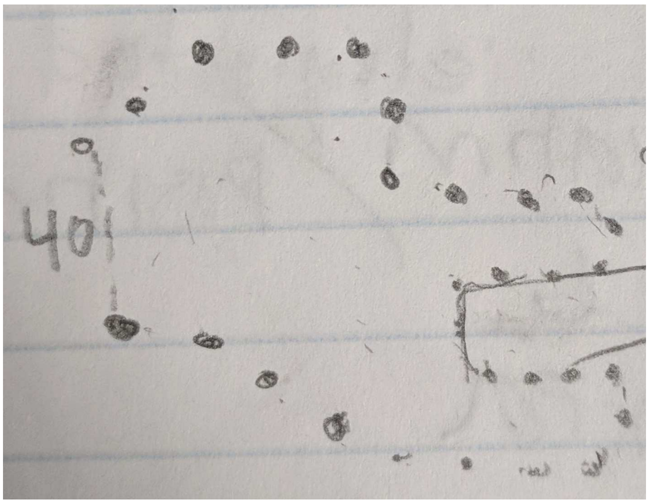

Figure 12: Drawing of a basic outline for the head.

Then, we added 17 extra points on the two outer outlines to define the head better. To do this, we drew a grid over the dragon's head and plotted points inside to make a nice fill. We then scaled it to be 80 ft wide and 65 ft tall so that it fitted into the side of the dragon's head. We positioned a drones in the formation given by this grid 5 ft away from both sides of the dragon's head.

0	1	0	0	0
1	1	1	0	0
1	1	1	1	1
1	1	1	1	0
0	0	1	1	0
0	0	0	1	1

Table 2: Grid for the Side of the head.

Lastly, we added an eye to each side, totaling 120 drones used for the head. We then had 80 left over drones. To make the tail come to a point, we used 37 drones. These drones followed the same sine wave function that the body drones followed. We then made the radius decrease from 20 ft to 0 ft over the course of the of the length of the tail. In order to follow the safety guideline of 5 ft between each drone, we had to use only one drone for the tip of the tale and only four drones for the penultimate drone circle.

Then, with 36 of the 43 leftover drones, we created a fire structure out of the mouth. The fire was made by making a copy of the tail and moving it towards the mouth. We then got rid of the very tip of the tale in order to make the fire look more realistic.

The next step was to design a color scheme for the dragon. We decided that the basic color for the dragon would be red since that is the most common color for a Chinese dragon. We decided to make the top-most drones yellow so that they looked like the spine. Next, we made the bottom three drones in each body circle white to make it look like a belly. We also made both the eyes white so that they would stand out. Finally we made the fire yellow so that it would stand out from the red color of the dragon. The final model can be seen in Figure 13.

The next step was determining how the dragon would be animated. We decide that, since the dragon is quite large, it would be easier for viewers if we only made the dragon oscillate in place. This meant that we only had to change the fundamental sine function of the dragon: $z = 10sin(x/10)$. The only new variable that this equation requires is time. We decided to use t to model time in seconds. The final equation is $z = 10sin(\frac{x}{10} + \frac{t\pi}{10})$. This means that the drones will move from their highest point to their lowest point in 10 seconds.

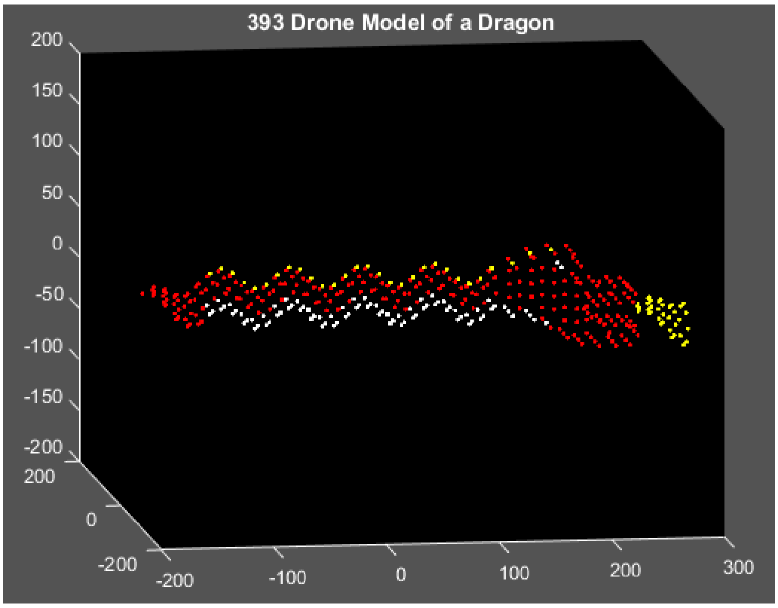

Figure 13: Model of the dragon using 393 drones

Then, we calculated where each of the drone groups should go on the dragon from the Ferris wheel. The groups are labeled below, and the groups will move in straight lines from their Ferris wheel positions to their dragon positions. The Ferris wheel and the dragon will have centers at the same point. The movement sequence is as follows: 3, 2, 9, 5, 6, 7, 1, 8, 4.

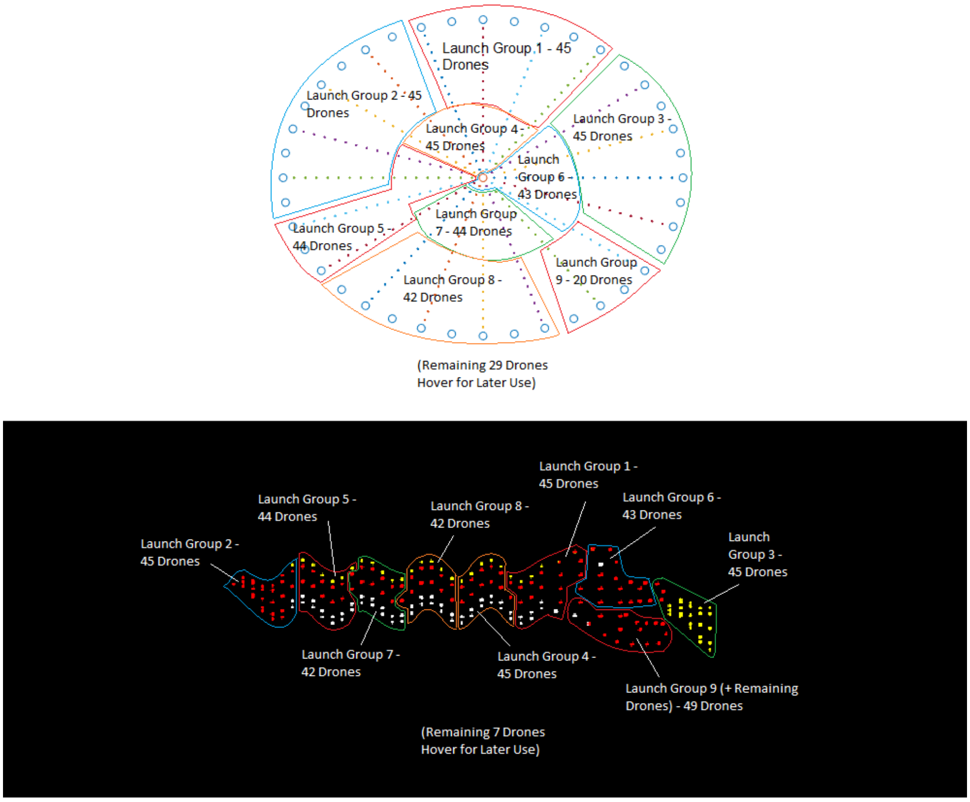

Figure 14: Drone Groups on the Ferris Wheel and the Dragon

7 Our Own Display: The Solar System

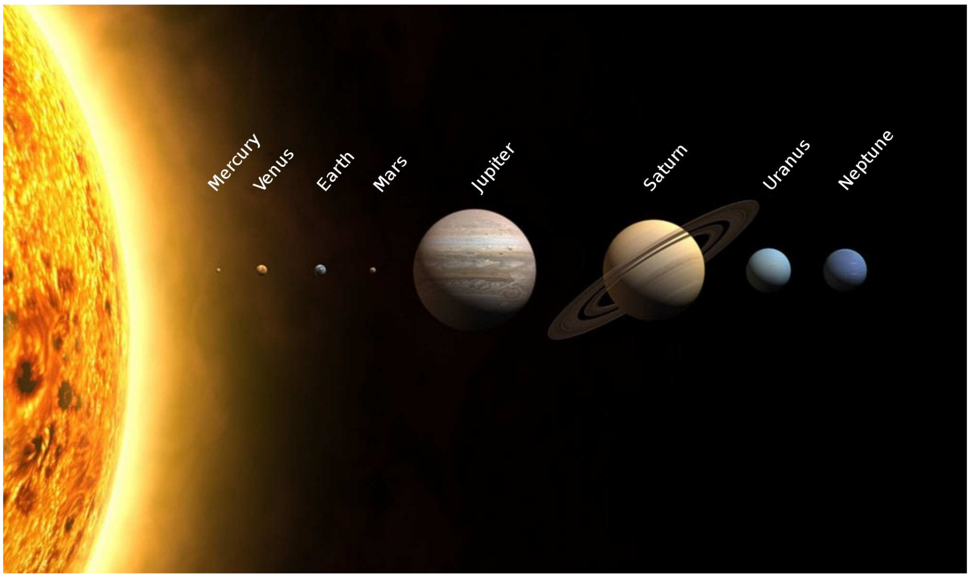

Figure 15: Diagram we will model our Solar System after(Source 5)

For the final display, we chose the solar system because it shows the individuality that the drones are capable of. The ferris wheel and the dragon show only showed the drones' capability of collaboration. It is also a concept that people of all intellects understand. Space is also a wondrous phenomenon, as is having a light show with drones.

First, we dispersed the drones into a model of the solar system- the sun, Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus and Neptune. Each of the planets will be formed by a series of drones in a spherical formation. The sun, we decided, would consist of 100 yellow drones in the center of the display. Mercury will be represented by 10 red drones. Venus will be represented by 28 white drones, and Earth will be represented by a 28 blue drones. Jupiter is shown by 52 orange drones. Saturn will be represented by 52 yellow drones with 20 white drones representing the rings of Saturn. Uranus and Neptune will each be represented by 42 drones, cyan for Uranus and blue for Neptune. The total drones used in this model is 396, with 4 drones remaining unlit. The drones will move in their planet formations orbiting around the sun. The sun's drones will remain hovering in their formation for the duration of the show.

For each planet we layered circle of different radii in order to form a hollow sphere. For example, Mars has 6 layers of circles containing 1 drone in the 1st layer, 4 drones in the 2nd layer, 6 drones in the 3rd layer, 6 drones in the 4th layer, 4 drones in the 5th layer, and 1 drone in the last layer. This spread the drones equally throughout the sphere.

The next step was to insure that each of the circles was the right size to properly define a sphere. We started by defining a constant step of $step = R/(rings - 1)$, where R is the radius of the sphere and rings is the total number of circles used to define that planet. Next, we found the distance in the y direction from a point on the circle to the center of the circle. This distance was given by the formula $d = abs(R - step(ringnumber - 1))$. The formula takes the radius of the circle and subtracts the distance that we have stepped towards it. If we have passed the center, the distance would come out negative, so we require an absolute value sign. From there, we can find the radius of a smaller circle through the formula $R^2 = r^2 + d^2$. Solving for r, we get the equation: $r = \sqrt{R^2 - d^2}$.

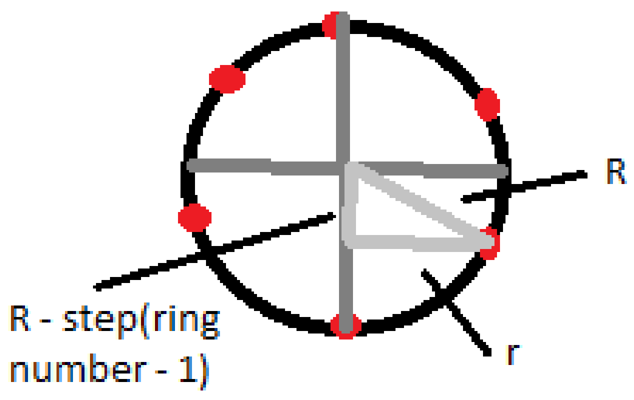

Figure 16: Trigonometry to determine the radius of the circle.

This model was used to make all of the planets, as well as the sun. Note that the solar system is not to scale, because then the sun would use 399 of the 400 drones.

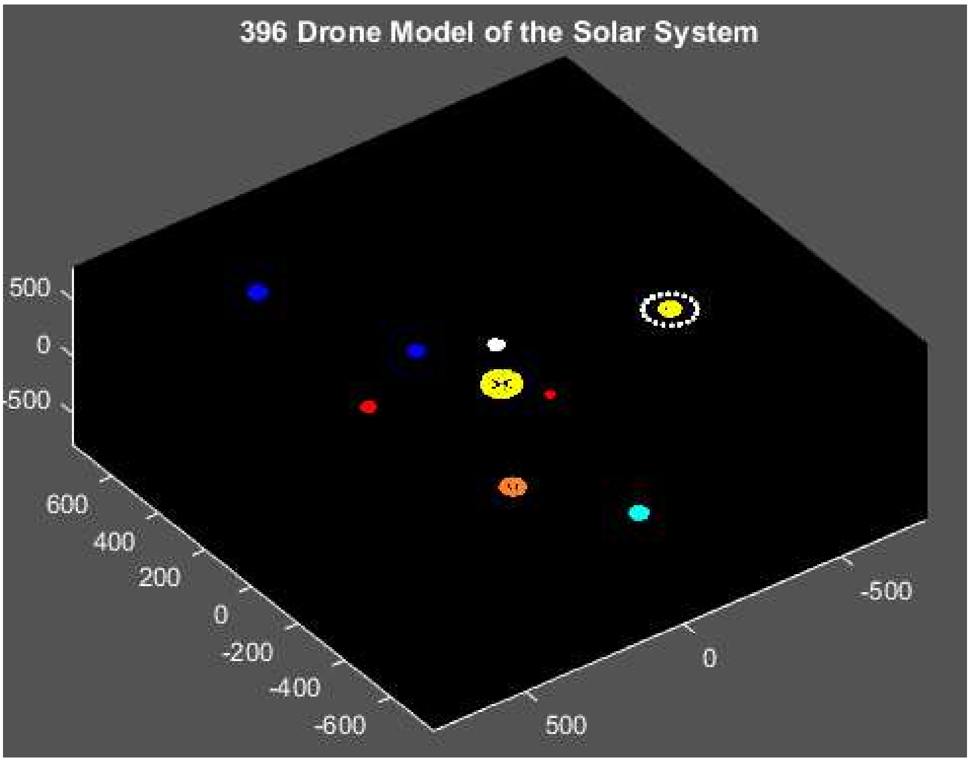

Figure 17: 396 Drone Solar System Model

The solar system display will last for three minutes. The planets will orbit the sun at the same speed, but since their orbit distances are different, the planets will be at varying positions as they revolve around the sun. Following the display of the solar system, there will be one minute allotted for the drones to safely return to the launch pad.

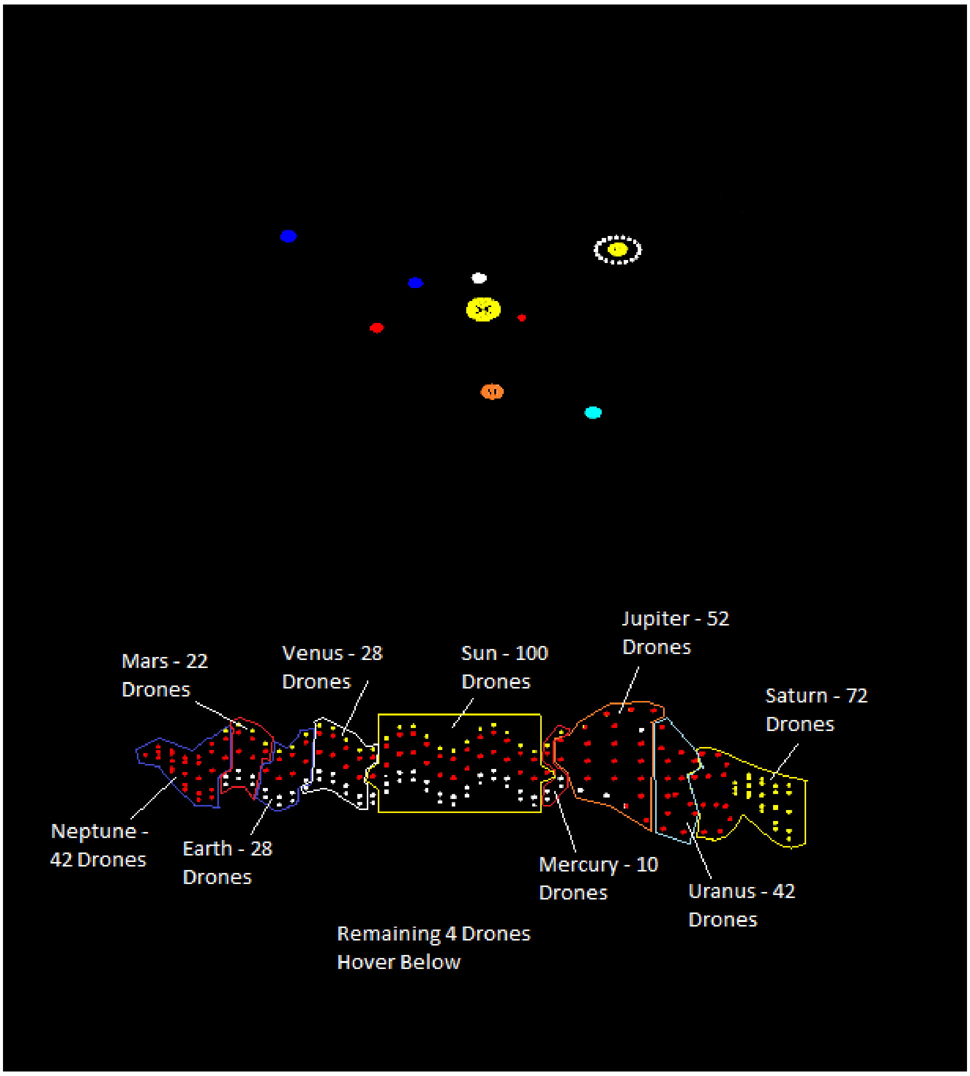

Figure 18: Groups to Travel from Dragon to Solar System

Instead of grouping the drones by launch group, this time we chose to group them by planet. This makes it much easier to get the drones to their position without colliding with each other.

8 Duration of the Show

Intel's drones are able to fly for roughly 20 minutes. With this in mind, we need to ensure that our light show is performed in a relatively quick and efficient manner to ensure that the drones come back to land well before there is a chance of drone failure.(Source 2)

We estimate that Intel's drones are able to fly a maximum of about 20 miles per hour, but we do not intend to go any faster than 15 miles per hour.

For the sake of the audience, we did not think a light show much longer than 10 minutes would be entertaining or visually pleasing. Since the drones are transforming into just three different shapes, we figured they would be in each formation (ferris wheel, dragon, solar system) for about 3 minutes. With the time taken for lift off, 2 transitions between displays, and landing, which would be about 1 minute each totaling 4 minutes, and landing and transitions, our show will last almost exactly 13 minutes. If the drones can fly for about 20 minutes, this gives us a few minutes of safety in case the drones are not all at full battery, or they lose battery while setting up the launch pad.

9 Air Space

In terms of airspace, we will require a volume of 350 feet of height by 1500 feet of width and length. This space will be centered around the launch zone. Since the dragon is about 500 feet long and the solar system will be of a similar diameter, we will give ourselves some extra space with 1500 feet in all directions. The ferris wheel, our tallest display, is 200 feet high, and with our displays being 100 feet above the ground, we have given ourselves a small bit of extra space above. The FAA limit for height of displays is currently 400 feet, so we are well within this limit.

10 Safety/Legality

For the safety of the audience, we plan to present our light show in such a way that, if some sort of failure does occur, anybody watching will be safe. Our primary safety concern is if a drone happens to lose power or control. We want to ensure that a falling drone would be well out of the way of our audience. Therefore, we plan to put the launch area well in front of the audience and have our show above the launch area rather than directly above the audience. This way, if a drone falls, it will fall back to the launch area instead of falling onto somebody watching the show. Similarly, putting the launch area some distance in front of the audience will ensure that any problems that could occur on launch or landing would not endanger anyone watching.

One issue we should not have to worry about is drones hitting each other and causing issues. Firstly, the drones have cages around the propellers, so if two drones did somehow manage to bump into each other they would most likely be fine. Secondly, because each drone is at least 5 feet away from each other drone, we have plenty of tolerance for drones being slightly out of position, so it is extremely unlikely that any drone would hit any other drone. In addition, the fact that the drones transition from display to display in their launch groups (rather than all flying individually in a straight line to a new point) greatly decreases the likelihood of any drones hitting one another.

In the event that a drone does fall on the audience, Intel's Shooting Star drone is quite safe. It is made of foam and flexible plastic, and has cages around the propellers.

The bottom drones in our show will be about 100 feet above the ground. This is high enough that the drones will safely clear any ground obstacles, such as power lines, but low enough that firstly, the drones will not fall from a great distance in the event of a failure, and secondly, that the drones will have no chance of interfering with anything airborne (planes, birds, etc.)

We also need to take weather into account. Intel's Shooting Star drones are designed to be able to fly in light rain. In terms of wind speed, we think that our show will be able to tolerate winds of up to about 5 miles per hour. Since our drones can fly at 20 miles per hour at a maximum, but we will only go 15 miles per hour, we reserve some power in our drones to be able to fly against the wind and still maintain speed such that the show will not be affected. Thus, as long as the show is held on a relatively mild day, the drones will have some tolerance to changes in weather.

In terms of legality, our show seems to meet all FAA regulations. We never go above 400 feet (the FAA's ceiling) and the FAA has approved Intel's drone shows being piloted by a single person. Thus, our show should be meet all legal requirements so long as we have approval from the city.(Source 3)

11 Cost

We assume that, based off of the prices of other drones, Intel's "Shooting Star" drones will cost around 200 dollars each (Intel has not released the price of the drone). Given this and the fact that we will use around 400 drones for our show, the price of the drones alone will be roughly 80,000 dollars. If we bought our own drones, we would probably also need to buy the program to run them from Intel, as well as training someone to pilot the drones. Additionally, there are some other operating costs, such as hiring workers to help set out drones or to perform safety inspections. In total, we can estimate that we will spend around 100,000 dollars if we buy our own drones. Of course, Intel may offer a program to rent their drones for a show. We can estimate that this would be somewhere around 20 percent of the cost to actually buy the drones. Thus, we can assume that we could spend as little as 16,000 dollars on drones. In this case, we would probably not have to buy a program from Intel, but there would still be similar operating costs to if we purchased the drones ourselves. Thus, we can assume that we would spend roughly 36,000 dollars if we were to rent drones. However, it is important to note that if we bought our own drones, we could run our own displays and have them multiple times, while if we rent drones, we have to pay a significant fee every time. Thus, in the long term, it is probably more cost effective to purchase our own drones, but in the short term, it is more cost effective to rent.

In comparison, most public fireworks displays range anywhere from 5,000 to 30,000 dollars. And the biggest Fourth of July fireworks display, the Macy's Fourth of July Fireworks Special, could cost around 6 million dollars for the fireworks alone.(Source 1, Source 4)

It is important to note that, while a drone show seems quite expensive in comparison to most fireworks shows, drones are reusable, while fireworks are not. Each drone could probably be used in up to hundreds of shows if it is properly taken care of. Thus, drone shows have a high initial price but become very appealing, financially speaking, in the long term.

12 Sensitivity Analysis

In general, our system is quite robust to changes if some of our assumptions are proven incorrect.

For example, if Intel's drones are significantly bigger on the ground than 1 foot, we can easily expand our launch system. It will be less efficient, but it will still work just fine. Similarly, if we find that Intel's drones require more than 5 feet between them, it would not be especially difficult to expand our displays. Proportions and number of drones would not have to change; only the size of the displays.

If, for some reason, the drones tend to block the light of other drones, expanding our display would likely fix this problem. If the drones were further apart, each drone would block a narrower arc of light from each other drone, decreasing the likelihood of light being blocked.

If we are wrong about the price of the drones, the only effect is that the cost for the city increases. While this isn't exactly ideal, it is by no means catastrophic to our plan.

If the flight time is shorter than 20 minutes, we can simply reduce the amount of time that the drones spend in each display. We have each display going for several minutes, so it would be very simple to cut down on display time.

If the drones are slower than we thought, we can reduce the time that each display is up to allow for more transition time, as well as slowing down each animation.

Overall, almost every aspect of our light display can be changed.

13 Conclusion

In conclusion, our team found that having the aerial light display at our annual festival would be beneficial to the city. The light show would add value and uniqueness to the festival, as well as impress the viewers. Overall, the show would draw more attention and popularity to the festival and the businesses involved.

For maximum cost efficiency and visual pleasure, our team found optimal designs of a ferris wheel, dragon and solar system. We concluded that a total of 400 drones would be sufficient for all of the displays.

For the design of the ferris wheel, we distributed almost all of the 400 drones amongst the outer circle, inner circle and 20 spokes. The drones in the ferris wheel will rotate in a clockwise path 6 times in 3 minutes. For the Chinese dragon display, we separated the drones between the body, tail, head and "flame breath." The dragon will appear to move in a wave for the 3 minutes allotted for its display. The solar system, split into the sun and 8 planets of variant size, is represented by drones in spherical formations. The planets will revolve around the sun in their respective orbits for the duration of the 3 minute show.

If considering long-term monetary effects of the drone use, buying the 400 drones would be the smartest option. The total cost of buying and operating the drones would be about 100,000 dollars. This price is significantly high, but it is a one-time purchase and the drones can be reused in future events. If short-term costs are important in the city's expenditures, renting would be the most effective option. The costs of renting would total at about 36,000 dollars, a price that would be repeated with each use of the drones.

14 Strengths and Weaknesses

Our system has quite a few strengths, one of which being that it is fairly robust to changes in our assumptions, as described in the section above.

Since we fly all our drones to exact points, our system provides both clear images and efficient use of drones.

If we had used a grid system of drones in the sky and just turned their lights on and off, we would have been extremely inefficient in terms of drone use.

Another strength is that we haven't used too many or too few drones. 400 is an effective number because it produces clear images without wasting drones and being excessively expensive.

A somewhat small weakness to our system is that our solar system model might be too big. It has a total diameter of about 1500 feet, so the drones will need to fly very far in order to get to their positions. This also significantly increases the airspace that our show will require.

However, our main weaknesses are that it is time-consuming to create our drone images and that it would be very difficult to change the number of drones in our existing images. To create a new display, we basically need to plot every point in MatLab (The program makes the process simpler, but it is still very time-consuming). If we were to change the number of drones in our existing images, it would be almost as difficult as creating an entirely new image. Ideally, we would be able to import a 3D model of what we want to create and give a program a number of drones, and have the computer create the display for us. This would make creating shows a much more streamlined and efficient process.

15 Sources Consulted

1. https://www.huffingtonpost.com/gobankingrates/4th-of-july-fireworks-the_b_7697388.html
2. https://www.engadget.com/2016/11/06/intel-shooting-star-drone/
3. https://www.engadget.com/2016/05/05/intel-faa-approved-drone-swarm-light-show/
4. http://www.foxbusiness.com/features/2012/06/27/explosive-costs-big-fireworks-displays.html
5. https://en.wikipedia.org/wiki/Solar_System#/media/File:Planets2013.svg
6. https://www.dronethusiast.com/best-drones-without-camera/
7. https://www.tes.com/lessons/f7GDMHIyMY4s5Q/chinese-dragons
8. https://www.bransontracks.com/rides/ferris-wheel/

16 Appendix

16.1 Code

```
%% Ferris Wheel
clear;clf;clc;
r = 20;
w = 0:(2*pi)/40:2*pi;
%for t = 1:1:60
 %   initial_pos_x = 1:r*cos(w*t):2*pi
  %  initial_pos_y = 1:r*sin(w*t):2*pi
   % movement = [r*cos(w*t),r*sin(w*t)];
```

```
%end

    xl = r*cos(w);
    dy = r*sin(w);
    scatter(xl,dy)
    hold on;
    %r2 = r/8:r/8:r-r/8;
    %w2 =  0:(2*pi)/20:2*pi;
    [r2,w2] = meshgrid(r/8:r/8:r-r/8, 0:(2*pi)/20:2*pi);
    x2 = r2.*cos(w2);
    y2 = r2.*sin(w2);
    plot(x2,y2,'.')
    scatter(0,0)

    axis([-25 25 -25 25])
    pause(0.1)

%% Dragon Body
clc;clf;clear;
xl= 0:1:12;

yl = xl-xl;

r = 20;

whitebg('black')
%sides
[t,xl] = meshgrid(3*pi/4:pi/4:pi,-120:10:130);
zl = 10*sin(xl/10);
dy = r*cos(t);
dz = r*sin(t);
dx = xl;
dy = dy;
dz = zl + dz;
plot3(dx,dy,dz,'r.')
hold on
```

```
[t,xl] = meshgrid(0:pi/4:pi/4,-120:10:130);
zl = 10*sin(xl/10);
dy = r*cos(t);
dz = r*sin(t);
dx = xl;
dy = dy;
dz = zl + dz;
plot3(dx,dy,dz,'r.')

%underbelly
[t,xl] = meshgrid(5*pi/4:pi/4:7*pi/4,-120:10:130);
zl = 10*sin(xl/10);
dy = r*cos(t);
dz = r*sin(t);
dx = xl;
dy = dy;
dz = zl + dz;
plot3(dx,dy,dz,'w.')

%spine
xl = -120:10:130;
zl = 10*sin(xl/10);
dy = 0*xl;
dz = r+(xl-xl);
dx = xl;
dy = dy;
dz = zl + dz;
plot3(dx,dy,dz,'y.')

%Tail
[t,xl] = meshgrid(0:pi/4:7*pi/4,-160:10:-130);
r = (2/5)*(xl+130)+20;
zl = 10*sin(xl/10);
dy = r.*cos(t);
dz = r.*sin(t);
dx = xl;
dy = dy;
```

```
dz = zl + dz;

plot3(dx,dy,dz,'r.')


[t,xl] = meshgrid(0:pi/2:7*pi/4,-170);

r = (2/5)*(xl+130)+20;

zl = 10*sin(xl/10);

dy = r.*cos(t);

dz = r.*sin(t);

dx = xl;

dy = dy;

dz = zl + dz;


plot3(dx,dy,dz,'r.')


plot3(-180,0,10*sin(-180/10),'r.')


% Dragon head


x1 = [28.1 44.1 62.5 79.3 85.7 84.9 100 115.3 127.3 133.7 131.3 120.1 109.7 100.1 95.3 106.5 116.1 125.

z1 = [22.5 12.9 9.7 10.5 22.5 38.5 41.7 42.5 43.3 50.5 58.5 59.3 60.1 60.9 72.9 80.1 80.9 80.9 81.7 91.

% Dragon Spine


x2 = [28.1 44.1 62.5];

z2 = [22.5 12.9 9.7];

z2 = -z2 + 51;

y2 = 0 + x2-x2;

x2 = x2 + 120;

plot3(x2,y2,z2,'y.')


% Dragon mouth


%Center

x2 = [79.3 85.7 84.9 100 115.3 127.3 133.7 131.3 120.1 109.7 100.1 95.3 106.5 116.1 125.7 132.9 136.1 1

z2 = [10.5 22.5 38.5 41.7 42.5 43.3 50.5 58.5 59.3 60.1 60.9 72.9 80.1 80.9 80.9 81.7 91.3 100.1 100.9

z2 = -z2 + 51;

y2 = 0 + x2-x2;

x2 = x2 + 120;
```

```
plot3(x2,y2,z2,'r.')

%Left
x2 = [28.1 44.1 62.5 79.3 85.7 84.9 100 115.3 127.3 133.7 131.3 120.1 109.7 100.1 95.3 106.5 116.1 125.
z2 = [22.5 12.9 9.7 10.5 22.5 38.5 41.7 42.5 43.3 50.5 58.5 59.3 60.1 60.9 72.9 80.1 80.9 80.9 81.7 91.
z2 = -z2 + 51;
y2 = 20 + x2-x2;
x2 = x2 + 120;
plot3(x2,y2,z2,'r.')

%Right
x2 = [28.1 44.1 62.5 79.3 85.7 84.9 100 115.3 127.3 133.7 131.3 120.1 109.7 100.1 95.3 106.5 116.1 125.
z2 = [22.5 12.9 9.7 10.5 22.5 38.5 41.7 42.5 43.3 50.5 58.5 59.3 60.1 60.9 72.9 80.1 80.9 80.9 81.7 91.
z2 = -z2 + 51;
y2 = -20 + x2-x2;
x2 = x2 + 120;
plot3(x2,y2,z2,'r.')


% White of dragon head

x2 = [58.5 44.1 24.9];
z2 = [ 81.7 73.7 69.7];
z2 = -z2 + 51;
y2 = 0 + x2-x2;
x2 = x2 + 120;
plot3(x2,y2,z2,'w.')

x2 = [ 44.1 24.9];
z2 = [ 73.7 69.7];
z2 = -z2 + 51;
y2 = -20 + x2-x2;
x2 = x2 + 120;
plot3(x2,y2,z2,'w.')

x2 = [44.1 24.9];
z2 = [73.7 69.7];
```

```
z2 = -z2 + 51;
y2 = 20 + x2-x2;
x2 = x2 + 120;
plot3(x2,y2,z2,'w.')

% Dragon head inside

%eye
plot3(70+120,10,-25+51, 'w.')
plot3(70+120,-10,-25+51, 'w.')



%side of head
x1 = [2 1 2 3 1 2 3 4 5 1 2 3 4 3 4 4 5];
z1 = [5 4 4 4 3 3 3 3 3 2 2 2 2 1 1 0 0];
dx = x1 * 20;
dz = z1 * 13;
y = -25 + dx - dx;
plot3(dx+130,y,dz-36, 'r.')
y = 25 + dx - dx;
plot3(dx+130,y,dz-36, 'r.')

axis([-200 300 -200 200 -200 200])
%Fire
[t,xl] = meshgrid(0:pi/4:7*pi/4,270:10:300);
r = (2/5)*(xl-300)+20;
zl = 10*sin(xl/10);
dy = r.*cos(t);
dz = r.*sin(t);
dx = xl;
dy = dy;
dz = zl + dz;
plot3(dx,dy,dz-25,'y.')

[t,xl] = meshgrid(0:pi/2:7*pi/4,260);
r = (2/5)*(xl-300)+20;
zl = 10*sin(xl/10);
```

```
dy = r.*cos(t);
dz = r.*sin(t);
dx = xl;
dy = dy;
dz = zl + dz;

plot3(dx,dy,dz-25,'y.')

%% Solar System
%Sun
theta = 5;
r1 = 0;
pos_x = r1*cos(theta);
pos_y = r1*sin(theta);

d = [4 8 10 12 16 16 12 10 8 4];
rings = 10;
R = 50;
step = (2*R)/(rings-1);
for ring = 1:1:rings
    r = sqrt(abs(R^2-(R-step*(ring-1))^2));
    t = 1:2*pi/d(ring):2*pi + 2*pi/d(ring);
    x = r*cos(t);
    y = r*sin(t);
    z = ring*step + x - x;
    plot3(pos_x+x,pos_y+y,z,'y.');
    hold on
end
%%

whitebg([0 0 0]);
% Mercury
theta = 300-pi/4;
r1 = 125;
pos_x = r1*cos(theta);
pos_y = r1*sin(theta);
d = [1 4 4 1];
```

```
rings = 4;
R = 7.5;
step = (2*R)/(rings-1);
for ring = 1:1:rings
    r = sqrt(abs(R^2-(R-step*(ring-1))^2));
    t = 1:2*pi/d(ring):2*pi + 2*pi/d(ring);
    x = r*cos(t);
    y = r*sin(t);
    z = ring*step + x - x;
    plot3(pos_x+x,pos_y+y,z,'r.');
    hold on
end
%%

%Mars
theta = 75+pi/2;
r1 = 350;
pos_x = r1*cos(theta);
pos_y = r1*sin(theta);
d = [1 4 6 6 4 1];
rings = 6;
R = 15;
step = (2*R)/(rings-1);
for ring = 1:1:rings
    r = sqrt(abs(R^2-(R-step*(ring-1))^2));
    t = 1:2*pi/d(ring):2*pi + 2*pi/d(ring);
    x = r*cos(t);
    y = r*sin(t);
    z = ring*step + x - x;
    plot3(pos_x+x,pos_y+y,z,'r.');
    hold on
end
%%

%venus
theta = 12792;
r1 = 190;
```

```
pos_x = r1*cos(theta);
pos_y = r1*sin(theta);

d = [1 5 8 8 5 1];
rings = 6;
R = 17;
step = (2*R)/(rings-1);
for ring = 1:1:rings
    r = sqrt(abs(R^2-(R-step*(ring-1))^2));
    t = 1:2*pi/d(ring):2*pi + 2*pi/d(ring);
    x = r*cos(t);
    y = r*sin(t);
    z = ring*step + x - x;
    plot3(pos_x+x,pos_y+y,z,'w.');
    hold on
end
%%

%earth
theta = 371;
r1 = 270;
pos_x = r1*cos(theta);
pos_y = r1*sin(theta);

d = [1 5 8 8 5 1];
rings = 6;
R = 17;
step = (2*R)/(rings-1);
for ring = 1:1:rings
    r = sqrt(abs(R^2-(R-step*(ring-1))^2));
    t = 1:2*pi/d(ring):2*pi + 2*pi/d(ring);
    x = r*cos(t);
    y = r*sin(t);
    z = ring*step + x - x;
    plot3(pos_x+x,pos_y+y,z,'b.');
    hold on
end
```

```
%%

%Jupiter
theta = 12+pi;
r1 = 440;
pos_x = r1*cos(theta);
pos_y = r1*sin(theta);

d = [1 4 8 8 10 8 8 4 1];
rings = 9;
R = 30;
step = (2*R)/(rings-1);
for ring = 1:1:rings
    r = sqrt(abs(R^2-(R-step*(ring-1))^2));
    t = 1:2*pi/d(ring):2*pi + 2*pi/d(ring);
    x = r*cos(t);
    y = r*sin(t);
    z = ring*step + x - x;
    plot3(pos_x+x,pos_y+y,z,'w.');
    hold on
end
%%

%Saturn
theta = 99;
r1 = 550;
pos_x = r1*cos(theta);
pos_y = r1*sin(theta);

d = [1 4 8 8 10 8 8 4 1];
rings = 9;
R = 25;
step = (2*R)/(rings-1);
for ring = 1:1:rings
    r = sqrt(abs(R^2-(R-step*(ring-1))^2));
    t = 1:2*pi/d(ring):2*pi + 2*pi/d(ring);
```

```
    x = r*cos(t);
    y = r*sin(t);
    z = ring*step + x - x;
    plot3(pos_x+x,pos_y+y,z,'y.');
    hold on
end

r = 70;
t = 1:2*pi/20:2*pi + 2*pi/5;
x = r*cos(t);
y = r*sin(t);
z = 25 + x - x;
plot3(pos_x+x,pos_y+y,z,'w.');
%%

%Uranus
theta = 201+pi;
r1 = 650;
pos_x = r1*cos(theta);
pos_y = r1*sin(theta);

d = [1 6 6 8 8 6 6 1];
rings = 8;
R = 20;
step = (2*R)/(rings-1);
for ring = 1:1:rings
    r = sqrt(abs(R^2-(R-step*(ring-1))^2));
    t = 1:2*pi/d(ring):2*pi + 2*pi/d(ring);
    x = r*cos(t);
    y = r*sin(t);
    z = ring*step + x - x;
    plot3(pos_x+x,pos_y+y,z,'c.');
    hold on
end
    %%

%Neptune
```

```
theta = 503;
r1 = 750;
pos_x = r1*cos(theta);
pos_y = r1*sin(theta);

d = [1 6 6 8 8 6 6 1];
rings = 8;
R = 20;
step = (2*R)/(rings-1);
for ring = 1:1:rings
    r = sqrt(abs(R^2-(R-step*(ring-1))^2));
    t = 1:2*pi/d(ring):2*pi + 2*pi/d(ring);
    x = r*cos(t);
    y = r*sin(t);
    z = ring*step + x - x;
    plot3(pos_x+x,pos_y+y,z,'b.');
    hold on
end
    axis([-1500 1500 -1500 1500 -1500 1500])
```